Time Domain Approach of the Ion Flow Field of Bipolar HVDC Transmission Lines

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Abstract-- In this paper, a highly stable time domain algorithm for analyzing the ion flow field of bipolar HVDC transmission lines is proposed. Usually, the steady state of this electric-field-space-charge coupled problem is directly solved and the transient process is ignored. However, the complete time domain analysis of this problem may provide a deep look at the formation process of DC ion flow field. In this work, the charge simulation method (CSM) is used to solve the space-charge-free electric field, the finite element method (FEM) is employed to solve the electric field in the presence of space charge, and the finite volume method (FVM) is applied to solve the current continuity equation. Well agreement is obtained between the calculation result and the experiment result.

I. INTRODUCTION

Space charges produced by corona phenomenon of HVDC transmission lines will form the ion flow field, which can influence the original space-charge-free electric field. The numerical simulation of ion flow field is difficult owing to its nonlinearity and often carried out directly in steady state with ignoring the transient process. [1]-[2] proposed an upwind FEM based on Kaptzov's assumption, which assumes that the electric field on coronating conductor surface maintains at the onset value. However, this method calculated the steady state directly and ignored the formation process of DC ion flow field. W. Li *et al*. proposed a dynamic simulation method based on the upwind FEM [3]. However, the time step of this method was restricted and it was only suitable to simulate a process with very short time such as surge corona. And this method only considered the unipolar condition.

This work proposes a time domain algorithm to solve the bipolar ion flow field completely from beginning to steady state. In order to solve the advection-recombination coupled problem and speed up the algorithm, strang operator splitting is used. The algorithm details are described at first. And then, the comparison of the calculation result with the experiment result is presented.

II. TIME DOMAIN ANALYSIS

A. Governing equations

The ion flow field of bipolar HVDC transmission lines is described as follows:

$$\nabla^2 \Phi(t) = -[\rho^+(t) - \rho^-(t)]/\varepsilon_0 \tag{1}$$

$$\boldsymbol{j}^+(t) = \rho^+(t)[\mu^+ \boldsymbol{E}(t) + \boldsymbol{W}(t)] \tag{2}$$

$$\boldsymbol{j}^-(t) = \rho^-(t)[\mu^- \boldsymbol{E}(t) - \boldsymbol{W}(t)] \tag{3}$$

$$\frac{\partial \rho^+(t)}{\partial t} = -\nabla \cdot \boldsymbol{j}^+(t) - R\frac{\rho^+(t)\rho^-(t)}{e} \tag{4}$$

$$\frac{\partial \rho^-(t)}{\partial t} = \nabla \cdot \boldsymbol{j}^-(t) - R\frac{\rho^+(t)\rho^-(t)}{e} \tag{5}$$

where, $\Phi(t)$ is the electric potential, $\rho^+(t)$ and $\rho^-(t)$ are the positive and negative charge densities, ε_0 is the permittivity of free space, $\boldsymbol{j}^+(t)$ and $\boldsymbol{j}^-(t)$ are the positive and negative ion current densities, μ^+ and μ^- are the ionic mobility of positive and negative ions, $\boldsymbol{E}(t)$ is the electric field intensity, $\boldsymbol{W}(t)$ is the wind velocity, R is the recombination coefficient, and e is the electron charge.

B. Process overview

The electric-field-space-charge coupled problem is solved iteratively by respectively solving the Poisson's equation (1) and current continuity equations (2)-(5). In each time step, the space-charge-free electric field is analyzed first using the CSM. Meanwhile the electric field in the presence of space charge, the value of which is taken as that of the last time step, is solved by the FEM. After obtaining the electric field intensity, the current continuity equations are solved to obtain the new space charge density by the FVM. Due to the couple of charge advection and recombination effect, strang operator splitting is used to decouple the problem. This process is carried out cyclically until reaching the required time.

C. Solution of the Poisson's equation

The electric potential $\Phi(t)$ can be divided into two parts, one of which is the space-charge-free potential $\varphi(t)$ and another is the electric potential $\psi(t)$ in the presence of space charge [3]. The space-charge-free potential $\varphi(t)$ due to the conductor voltage can be easily calculated by the charge simulation method. The electric potential $\psi(t)$ in the presence of space charge can be solved by the traditional finite element method. Typical triangle mesh and one order shape function is used. The electric potential at the node and the average electric field intensity at the center of the triangle can be obtained.

D. Solution of the current continuity equation

Solving the current continuity equation in time domain is the most difficult part of this problem. From (2)-(5), we can obtain the following two equations:

$$\frac{\partial \rho^+(t)}{\partial t} = -\nabla \cdot [\rho^+(t)\boldsymbol{V}^+(t)] - R\frac{\rho^+(t)\rho^-(t)}{e} \tag{6}$$

$$\frac{\partial \rho^-(t)}{\partial t} = -\nabla \cdot [\rho^-(t)\boldsymbol{V}^-(t)] - R\frac{\rho^+(t)\rho^-(t)}{e} \tag{7}$$

where, $\boldsymbol{V}^+(t)=\mu^+\boldsymbol{E}(t)+\boldsymbol{W}(t)$ and $\boldsymbol{V}^-(t)=-\mu^-\boldsymbol{E}(t)+\boldsymbol{W}(t)$.

Integrating (6) in the ith element and using Green's formula, follow equation can be obtained:

$$\int_i \frac{\partial \rho^+(t)}{\partial t} ds = -\int_i \rho^+(t)\boldsymbol{V}^+(t)\cdot \boldsymbol{n}_i dl - \int_i R\frac{\rho^+(t)\rho^-(t)}{e} ds \tag{8}$$

where $\boldsymbol{n}_i$ is the unit outward normal vector of element i. The charge density ρ is laid at the center of each element and considered to be uniformly distributed in each element. Then the integration can be simplified as:

$$\frac{\partial \rho^{+}(t)}{\partial t} S_i = -\sum_{x=j,k,m} \rho_{ix}^{+}(t)\boldsymbol{V}_{ix}^{+}(t)\cdot \boldsymbol{n}_{ix} L_{ix} - R\frac{\rho_i^{+}(t)\rho_i^{-}(t)}{e} S_i \quad (9)$$

where, S_i is the area of the ith element, $\rho_{ix}^{+}(t)$ is the charge density of the edge between element i and x, $\boldsymbol{V}_{ix}^{+}(t)$ is the corresponding velocity of the edge, and L_{ix} is the edge length. Upwind method is introduced to calculate the edge charge density $\rho_{ix}^{+}(t)$ as follows [4]:

$$\rho_{ix}^{+}(t) = \begin{cases} \rho_i^{+}(t), & if \ \boldsymbol{V}_{ix}^{+}(t)\cdot \boldsymbol{n}_{ix} > 0 \\ \rho_x^{+}(t), & if \ \boldsymbol{V}_{ix}^{+}(t)\cdot \boldsymbol{n}_{ix} < 0 \end{cases} \quad (10)$$

The edge velocity $\boldsymbol{V}_{ix}^{+}(t)$ can be obtained by interpolation as shown in Fig. 1:

$$\boldsymbol{V}_{ix}^{+}(t) = \boldsymbol{V}_i^{+}(t)\frac{d_{xi}}{d_{xi}+d_{ix}} + \boldsymbol{V}_x^{+}(t)\frac{d_{ix}}{d_{xi}+d_{ix}} \quad (11)$$

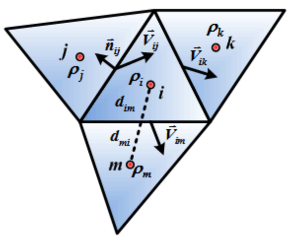

Fig. 1 Calculation of the charge density

Similarly, we can obtain the equation of the negative charges:

$$\frac{\partial \rho^{-}(t)}{\partial t} S_i = -\sum_{x=j,k,m} \rho_{ix}^{-}(t)\boldsymbol{V}_{ix}^{-}(t)\cdot \boldsymbol{n}_{ix} L_{ix} - R\frac{\rho_i^{+}(t)\rho_i^{-}(t)}{e} S_i \quad (12)$$

E. Strang operator splitting

The typical backward Euler method is unstable if the time step is large, which will restrict the simulation speed [3]. In this paper, the Crank-Nicolson method is used [5]. In order to solve the advection-recombination coupled equations (9) and (12), strang operator splitting is applied, which can decouple the nonlinear problem and speed up the algorithm.

Decouple the equations into two independent ones as:

$$\begin{cases} \dfrac{\partial \rho_i^{+}(t)}{\partial t} = -R\dfrac{\rho_i^{+}(t)\rho_i^{-}(t)}{e} \\ \dfrac{\partial \rho_i^{-}(t)}{\partial t} = -R\dfrac{\rho_i^{+}(t)\rho_i^{-}(t)}{e} \end{cases} \quad (13)$$

$$\begin{cases} \dfrac{\partial \rho_i^{+}(t)}{\partial t} = -\sum\limits_{x=j,k,m} \rho_{ix}^{+}(t)\boldsymbol{V}_{ix}^{+}(t)\cdot \boldsymbol{n}_{ix} L_{ix} \big/ S_i \\ \dfrac{\partial \rho_i^{-}(t)}{\partial t} = -\sum\limits_{x=j,k,m} \rho_{ix}^{-}(t)\boldsymbol{V}_{ix}^{-}(t)\cdot \boldsymbol{n}_{ix} L_{ix} \big/ S_i \end{cases} \quad (14)$$

While separating these equations in time domain, the Crank-Nicolson method is taken:

$$\frac{\partial \rho(t)}{\partial t} = \frac{\rho(t+\Delta t)-\rho(t)}{\Delta t} \quad (15)$$

$$\rho^{*}(t) = \frac{\rho(t)+\rho(t+\Delta t)}{2} \quad (16)$$

In the time step n, the nonlinear equation (13) is solved first with taking the time step as $\Delta t/2$ and the initial value as $\rho(n)$. Newton-Raphson iteration is carried out in each element and the charge density $\rho'(n)$ is obtained. Then, the linear equation (14) is solved with taking the time step as Δt and the initial value as $\rho'(n)$. The edge velocity is calculated by using the electric field intensity of the last time step and the charge density $\rho''(n)$ is obtained. Finally, (13) is solved again with taking the time step as $\Delta t/2$ and the initial value as $\rho''(n)$, then the new charge density $\rho(n+1)$ of the next time step can be obtained.

F. The steady state

The time-independent method usually takes the conductor surface electric field intensity as the convergence condition. However, in the time domain approach, when the conductor surface electric field is nearly stable, the space charges even haven’t reach the ground due to that the conductor surface electric field intensity is slightly affected by the space charges that far away from the conductor. If the ion flow current on the ground is mainly concerned, the algorithm terminates when the ion flow current on the ground is stable.

III. VALIDATION

The calculation result is compared with the experiment result of the ±800 kV HVDC test lines in the National Engineering Laboratory for UHV Technology (Kunming) of China Southern Power Grid. The configuration of the test line is shown in Fig. 2. In the experiment, the ion flow current on the ground is measured under a wind velocity of 0-3 m/s blowing from the negative polar to the positive polar. The calculated steady state result matches the measured result well.

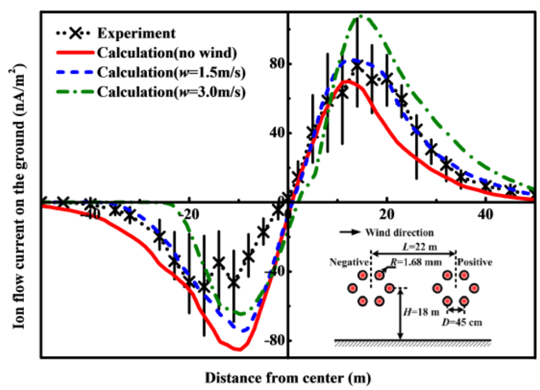

Fig. 2 Comparison of the calculated ion flow current with the experiment

The formation process of DC ion flow field will be presented in the full paper due to the limit of the 2 page digest.

IV. REFERENCES

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